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1989 J. Phys.: Condens. Matter 1 5531

(http://iopscience.iop.org/0953-8984/1/32/024)

J. Phys.: Condens. Matter 1 (1989) 5531–5534. Printed in the UK

LETTER TO THE EDITOR

Semiclassical scattering corrections to the quantum Hall effect conductivity and resistivity tensors

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Received 17 March 1989

Abstract. Ando, Matsumoto and Uemura published an important paper in 1975 that greatly influenced the early experimental work on the quantum Hall effect. Their paper showed that, in both a semiclassical scattering model and in a self-consistent Born approximation, there is a correction to the quantum Hall conductivity component σ_{xy} of the conductivity tensor that is directly proportional to the diagonal conductivity component σ_{xx}. We provide a detailed derivation of their results using the semiclassical scattering (relaxation-time approximation) model. We then present the surprising result that, in the semiclassical scattering model, there is no correction to the quantum Hall resistivity tensor component ρ_{xy} due to a finite value of ρ_{xx}.

In 1975, Ando, Matsumoto and Uemura published a paper [1] on the theory of the Hall effect for two-dimensional electron systems. They presented equations, obtained from a semiclasical scattering model, which had a first-order correction to the Hall conductivity component σ_{xy} of the conductivity tensor due to a finite value of the diagonal conductivity component σ_{xx}. That paper stimulated much of the early experimental work on the quantum Hall effect [2–4]. We use the semiclassical scattering or relaxation-time approximation model to show how Ando and co-workers obtained their results. We then report the surprising and little-known result that in the semiclassical model there is no correction to the quantum Hall effect resistivity tensor component ρ_{xy} due to a finite value of the longitudinal resistivity tensor component ρ_{xx}.

The quantum Hall effect requires the existence of a two-dimensional sheet of conducting charges. Let this sheet of charged particles be located in the xy plane. Assume that each charge q has an effective mass m^*. These charges will usually be electrons with charge $q = -e$.

Apply a strong magnetic field $\boldsymbol{B}$ perpendicular to the two-dimensional system. Let this field point in the positive z direction; i.e. $\boldsymbol{B} = B_z\hat{z}$. Initially, assume the charges are free particles. Each particle will execute cyclotron motion about its centre of mass. This motion will be in the xy plane. The equation of motion about the centre of mass of each particle is $\boldsymbol{F} = m\boldsymbol{a}$ or $q\boldsymbol{v}_c \times \boldsymbol{B} = m^*\,\mathrm{d}\boldsymbol{v}_c/\mathrm{d}t$, where $\boldsymbol{v}_c$ is the cyclotron velocity. The charges undergo simple harmonic motion about the centre of mass with angular frequency $\omega_c = qB_z/m^*$ and radius $r_c = v_c/\omega_c$. The cyclotron motion is quantised into Landau orbitals of total energies $E_N = (N + \frac{1}{2})\hbar\omega_c$, where $N = 0, 1, 2$ etc. The total

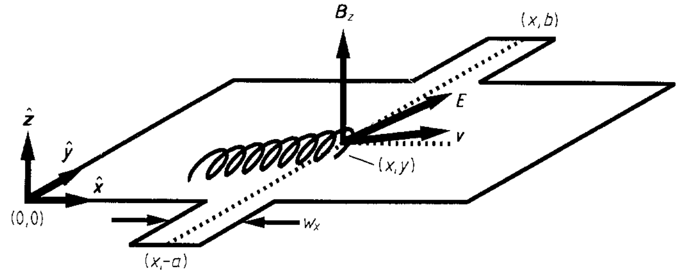

Figure 1. Cycloidal motion of charge q located at point (x, y) at time t. The motion is in the xy plane. There is an externally applied magnetic field $\boldsymbol{B}$ in the z direction. The charge has an effective mass m^*, a centre-of-mass drift velocity $\boldsymbol{v}$, and is in an induced electric field $\boldsymbol{E}$. The velocity $\boldsymbol{v}$ need not point in the x direction, the electric field $\boldsymbol{E}$ need not point in the y direction, and $\boldsymbol{E}$ need not be perpendicular to v. Hall-voltage-potential probes are located on the edge of the two-dimensional plane at points $(x, -a)$ and (x, b). Points $(x, -a)$ and (x, b) lie on a line in the y axis through point (x, y).

energies are equal to the kinetic energies because the charges are free particles. Therefore $\frac{1}{2}\hbar\omega_c = \frac{1}{2}m^* v_c^2$ and $r_c = (\hbar/qB_z)^{1/2}$ for charges occupying the lowest energy Landau level $N = 0$.

Now assume that the particles have drift velocities $\boldsymbol{v}$ in the xy plane, as shown in figure 1. The velocity of every particle is constant; but the magnitude and direction of the velocity can be different for each particle. The x axis of the figure is chosen to be the direction that $\boldsymbol{v}$ would have if there were no magnetic field and no scattering.

There is, however, a magnetic field. Therefore, the particles make cycloidal paths in the xy plane, and the velocity of each particle no longer necessarily points in the x direction. The cycloidal motion can be separated into the cyclotron motion about the centre of mass, which was discussed above, and the drift motion of the centre of mass.

We next discuss the drift motion of the centre of mass. Let a particle at point (x, y) be confined within a region of finite width w, where $w = a + b$ between points $(x, -a)$ and (x, b) of figure 1. Points $(x, -a)$ and (x, b) lie along the y axis of a quantum Hall sample, and therefore are places where Hall voltages can be measured. Three forces act upon the centre of mass of the particle: (i) the Lorentz force $q\boldsymbol{v} \times \boldsymbol{B}$; (ii) the electrical force $q\boldsymbol{E}$ due to mutual Coulomb repulsion; and (ii) a velocity-dependent frictional force or viscous-damping $-m^*\, \boldsymbol{v}/\tau$, where τ is the average time between scatterings, i.e. the relaxation time. The scattering time is very long compared with the cyclotron period. The magnetic field $\boldsymbol{B}$ is perpendicular to $\boldsymbol{E}$ and to $\boldsymbol{v}$, but $\boldsymbol{E}$ is not necessarily perpendicular to $\boldsymbol{v}$. The electric field $\boldsymbol{E}$ need not be the same for each particle because $\boldsymbol{v}$ is not necessarily the same. Also, E_x need not be zero; therefore, the Hall angle $\theta_H = \tan^{-1}(E_y/E_x)$ can be less than 90°.

The equation of motion of each centre of mass is $\boldsymbol{F} = m\boldsymbol{a}$, or

$$q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} - m^*\, \boldsymbol{v}/\tau = m^*\, \mathrm{d}\boldsymbol{v}/\mathrm{d}t = 0. \qquad (1)$$

One obtains from (1) that

$$v_x = (q\tau/m^*)\{1/[1 + (\omega_c\tau)^2]\}[E_x + (\omega_c\tau)E_y] \qquad (2)$$

and

$$v_y = (q\tau/m^*)\{1/[1 + (\omega_c\tau)^2]\}[E_y - (\omega_c\tau)E_x]. \qquad (3)$$

The charged particles form a conducting medium in which $\boldsymbol{j} = \tilde{\sigma}\boldsymbol{E}$. The current density $\boldsymbol{j}$ has dimensions of current per unit width rather than current per unit area because the medium is two-dimensional. Also, for the same reason, the components of the conductivity tensor $\tilde{\boldsymbol{\sigma}}$ have dimensions of $(\Omega)^{-1}$ rather than $(\Omega\ \mathrm{m})^{-1}$.

Now $j_x = \sigma_{xx}E_x + \sigma_{xy}E_y$. But $\boldsymbol{j} = nq\boldsymbol{v}$, where n is the number of conducting charges per unit area of the two-dimensional surface and is not necessarily constant over the surface. Therefore, $j_x = nqv_x = \sigma_{xx}E_x + \sigma_{xy}E_y$ and $j_y = nqv_y = \sigma_{yx}E_x + \sigma_{yy}E_y$. Using (2) and (3) for v_x and v_y, one obtains the equations

$$\sigma_{xx} = \sigma_{yy} = (nq/B_z)\{(\omega_c\tau)/[1 + (\omega_c\tau)^2]\} \tag{4}$$

and

$$\sigma_{xy} = -\sigma_{yx} = (nq/B_z)\{(\omega_c\tau)^2/[1 + (\omega_c\tau)^2]\} = (\omega_c\tau)\sigma_{xx} = nq/B_z - \sigma_{xx}/\omega_c\tau. \tag{5}$$

These are the equations obtained by Ando and co-workers [1].

Let the conducting charges be electrons, so $q = -e$. Adjust the magnetic field at a quantum Hall step so that the electrons occupy all the allowed states of i quantum levels but no states of other quantum levels. There are at least two quantum levels per Landau level because the electrons undergo a spin-splitting energy difference in the presence of magnetic fields. It then follows that $n = i\,eB_z/h$, and the factor nq/B_z in equations (4) and (5) becomes $-e^2i/h$. The above result from the semiclassical or relaxation-time approximation model then predicts that there is a first-order correction to σ_{xy} due to a finite value of σ_{xx}.

One can obtain the resistivity tensor $\tilde{\boldsymbol{\rho}}$ from the equations $\boldsymbol{E} = \tilde{\rho}\boldsymbol{j}$ and $\boldsymbol{j} = nq\boldsymbol{v}$ using the same method as in the last section, but it is easier to simply invert the conductivity tensor $\tilde{\boldsymbol{\sigma}}$ in order to obtain $\tilde{\boldsymbol{\rho}}$

$$\rho_{xx} = \rho_{yy} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{yy}^2) \tag{6}$$

and

$$\rho_{xy} = -\rho_{yx} = -\sigma_{xy}/\sigma_{xx}\ \rho_{xx}. \tag{7}$$

Using (4) and (5), we obtain the equations

$$\rho_{xx} = \rho_{yy} = (B_z/nq)[1/(\omega_c\tau)] = -(h/e^2i)[1/(\omega_c\tau)] \tag{8}$$

and

$$\rho_{xy} = -\rho_{yx} = -(\omega_c\tau)\rho_{xx} = -B_z/nq = h/e^2i \tag{9}$$

if $n = i\,eB_z/h$ and $q = -e$. Note that $\rho_{xy} = h/e^2i$, even when $\boldsymbol{v}$ is not in the x direction and even when the Hall angle, θ_H is not 90°.

We thus find the surprising result that in the semiclassical scattering or relaxation-time approximation model there is no direct correction to the quantum Hall resistivity tensor component ρ_{xy} due to a finite value of ρ_{xx}, whereas there is a first-order correction to the quantum Hall conductivity tensor component σ_{xy} due to a finite value of σ_{xx}. This result is due to the two-dimensionality of the resistivity tensor, and remains valid for all scattering mechanisms that can be expressed in terms of a velocity-dependent viscous-damping force $-m^*\,\boldsymbol{v}/\tau$. In such scattering mechanisms $\sigma_{xy} = (\omega_c\tau)\sigma_{xx}$ and $\rho_{xy} = -(\omega_c\tau)\rho_{xx}$.

Of course, even when $\rho_{xy} = -(\omega_c\tau)\rho_{xx}$, there can stil be a correction to ρ_{xy} due to scattering of conducting particles out of filled quantum levels because $n \neq i\,eB_z/h$.

These semiclassical scattering model predictions are for quantum Hall devices of infinite length. Rendell and Girvin [5] and von Klitzing and co-workers [6] predict that there is a geometrical correction to ρ_{xy} due to non-zero values of ρ_{xx} for devices with finite length-to-width ratios. This geometrical correction arises from the fact that the current enters and exits at opposite corners of the devices.

Linear corrections to the quantised Hall resistivity ρ_{xy} due to finite values of ρ_{xx} have been observed in temperature dependence experiments [7, 8]. However, as pointed out by van der Wel and co-workers [9], these corrections may actually be due to finite widths of the Hall potential probes, rather than an inherent physical property of the two-dimensional system. If the Hall potential probes at points $(x, -a)$ and (x, b) in figure 1 have finite widths w_x in the x direction, then equipotential curves concentrate at opposite corners of those probes. Therefore, any amount of longitudinal voltage drop along the probe width w_x contributes directly to the Hall voltage, and ρ_{xy} thus appears to be linearly dependent upon ρ_{xx}.

How general are these semiclassical model results? Ando and co-workers [1] showed that short-range scattering in a self-consistent Born approximation is an example of a scattering mechanism that can be expressed in terms of a velocity-dependent damping force. It would be very useful to investigate if other scattering mechanisms can also be expressed in this form, especially those for real devices at finite temperatures.

The author thanks S M Girvin of Indiana University for his useful conservations, and B N Taylor of NIST for his encouragement. This work was partially supported by the Calibration Coordination Group of the Department of Defense, the Naval Strategic Systems Programs Office, and by Sandia National Laboratories.

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